CHAPTER 16
History of Logic

Logic was born in ancient Greece and reborn a century ago. Logic keeps growing and expanding, and has contributed to the birth of the computer age. We can better understand and appreciate logic by studying its history.

16.1 Ancient logic

The formal study of valid reasoning began with Aristotle (384–322 BC) in ancient Greece. An unprecedented emphasis on reasoning prepared for Aristotle's logic. Greeks used complex reasoning in geometry, to prove results like the Pythagorean theorem. Sophists taught rich young men to gain power by arguing effectively (and often by verbal trickery). Parmenides and Heraclitus reasoned about being and non-being, anticipating later disputes about the law of non-contradiction, and Zeno reasoned about paradoxes. Socrates and Plato gave models of careful philosophical reasoning; they tried to derive absurdities from proposed views and sought beliefs that could be held consistently after careful examination.

Reasoning is an important human activity, and it didn't begin in ancient Greece. Is this ability biologically based, built into our brains by evolution because it aids survival? Or does it have a divine origin, since we're made in the "image and likeness" of God? Or do both explanations have a place? Logic raises fascinating issues for other disciplines.

Aristotle began the *study* of logic. He was the first to formulate a correct principle of inference, to use letters for terms, and to construct an axiomatic system. He created syllogistic logic (Chapter 2), which studies arguments like these (using "all A is B," "no A is B," "some A is B," or "some A is not B"):

Valid argument	➔	All humans are mortal.	all H is M
		All Greeks are humans.	all G is H
		∴ All Greeks are mortal.	∴ all G is M

This is *valid* because of its formal structure, as given by the formulation on the right; any argument having this same structure will be valid. If we change the structure, we may get an invalid argument, like this one:

Invalid argument	➔	All Romans are mortal. All Greeks are mortal. ∴ All Greeks are Romans.	all R is M all G is M ∴ all G is R

This is *invalid* because its form is wrong. Aristotle defended valid forms by deriving them from self-evidently valid forms; he criticized invalid forms by showing that they sometimes give true premises and a false conclusion. His logic of syllogisms is about *logic in a narrow sense,* since it deals with what follows from what. He also pursued other topics that connect with appraising arguments, such as definitions and fallacies; these are about *logic in a broader sense.*

Aristotle proposed two principles of thought. His **law of non-contradiction** states that the same property cannot at the same time both belong and not belong to the same object in the same respect. So "S is P" and "S is not P" can't both be true at the same time, unless we take "S" or "P" differently in the two statements. Aristotle saw this law as so certain that it can't be proved by anything more certain; not all knowledge can be demonstrated, since otherwise we'd need an infinite series of arguments that prove every premise by a further argument. Deniers of the law of non-contradiction assume it in their practice; to drive this point home, we might bombard them with contradictions until they plead for us to stop. Aristotle also supported the **law of excluded middle**, that either "S is P" or "S is not P" is true. Some deviant logics today dispute both laws (Chapter 17).

Aristotle also studied the logic of "necessary" and "possible" (see *modal logic,* Chapters 10 and 11). He discussed future contingents (events that may or may not happen). Consider a possible sea battle tomorrow. If "There *will* be a sea battle tomorrow" ("S" below) is *now* either true or false, this seems to make necessary whether the battle occurs:

Either it's true that S or it's false that S.
If it's true that S, then it's necessary that S.
If it's false that S, then it's necessary that not-S.
∴ Either it's necessary that S or it's necessary that not-S.

Aristotle rejected the conclusion, saying that there was no necessity either way. He seemed to deny the first premise and thus the universal truth of the law of excluded middle (which he elsewhere defends); if we interpret him this way, then he anticipated many-valued logic in using a third truth value besides *true* and *false* (§17.1). Another solution is possible. Many think premises 2 and 3 have a box-inside/box-outside ambiguity (§10.1): taking them as "$(A \supset \Box B)$" makes them doubtful while taking them as "$\Box(A \supset B)$" makes the argument invalid.

After Aristotle, Stoics and others developed a logic that focused on "if-then," "and," and "or," like our propositional logic (Chapters 6 and 7). Stoic logicians defended, for example, an important inference form that came to be called *modus tollens* (denying mode):

Valid argument	➔	If your view is correct, then such and such is true. Such and such is false. ∴ Your view isn't correct.	If C then S Not-S ∴ Not-C

Stoics also studied modal logic. Unlike logicians today, they took "necessary" and "possible" in a temporal sense, like "true at all times" and "true at some times." They disputed whether there was a good modal argument for *fatalism*, the view that all events happen of inherent necessity (see §10.3b #10). They also disputed how to understand "If A then B" (§17.4). Philo of Megara saw it as true if and only if it's not *now* the case that A is true and B is false; this fits the modern truth table for "if-then." Diodorus Chronos saw it as true if and only if A is *never* at any time true while B is false.

Aristotelian and Stoic logic were first seen as rivals, differing in three ways:

- Aristotle focused on "all," "no," and "some." Stoics focused on "if-then," "and," and "or."
- Aristotle used letter variables and expressed arguments as long conditionals, like "If all A is B, and all C is A, then all C is B." Stoics used number variables and expressed arguments as sets of statements, like "If 1 then 2. But not-2. Therefore, not-1."
- Aristotle saw logic not as part of philosophy but rather as a tool for all thinking. Stoics saw logic as one of philosophy's three branches (the other two being physics and ethics). But both agreed that students should study logic early, before going deeply into other areas.

Later thinkers combined these approaches into **traditional logic**. For the next two thousand years, Aristotle's logic with Stoic additions ruled in the West.

At the same time, another tradition of logic rose up in India, China, and Tibet. We call it **Buddhist logic** even though Hindus and others pursued it too. It studied many topics important in the West, including inference, fallacies, and language. This is a common pattern in Buddhist logic:

Here there is fire, because there is smoke.
Wherever there is smoke there is fire, as in a kitchen.
Here there is smoke.
∴ Here there is fire.

The last three lines are deductively valid:

All cases of smoke are cases of fire.
This is a case of smoke.
∴ This is a case of fire.

This omits "as in a kitchen," which suggests inductive reasoning (Chapter 5); in our experience of smoke and fire, smoke always seems to involve fire.

The Eastern logic tradition is poorly understood in the West; this tradition covers many centuries, and many texts are difficult or untranslated. Some commentators emphasize similarities between East and West; they see human thinking as essentially the same everywhere. Others emphasize differences and caution against imposing a Western framework on Eastern thought. And some deviant logicians see the Eastern tradition as congenial to their views.

Many see the East as more mystical than logical; Zen Buddhism delights in using paradoxes (like the sound of one hand clapping) to move us beyond logical thinking toward a mystical enlightenment. But East and West both have logical and mystical elements. Sometimes these come together in the same individual; Ludwig Wittgenstein in the early 20th century invented truth tables but also had a strongly mystical side.

16.2 Medieval logic

Medieval logicians carried on the basic framework of Aristotle and the Stoics, as logic became important in higher education.

The Christian thinker Boethius (480–524) helped the transition to the Middle Ages. He wrote on logic, including commentaries; he explained the modal box-inside/box-outside ambiguity as he defended the compatibility of divine foreknowledge and human freedom (§10.3b #4 and #14). He translated Aristotle's logic into Latin. Many of his translations were lost; but his *Categories* and *On Interpretation* became the main source for the *logica vetus* (old logic).

The Arab world dominated in logic from 800–1200. Some Arab logicians were Christian, but most were Muslim; both groups saw logic as important for theology and medicine. They translated Aristotle into Arabic and wrote commentaries, textbooks, and original works. They pursued topics like syllogisms, modal logic, conditionals, universals, predication, and existence. Baghdad and Moorish Spain were centers of logic studies.

In Christian Europe, logic was reborn in the 11th and 12th centuries, with Anselm, Peter Abelard, and Latin translations of Aristotle's *Prior Analytics*, *Posterior Analytics*, *Topics*, and *Sophistical Refutations*; the *logica nova* (new logic) was based on these. There was interest in universals and in how terms signify. Peter of Spain and William of Sherwood wrote logic textbooks.

The clever Barbara-Celarent verse was a tool for teaching syllogisms:

> Barbara, Celarent, Darii, Ferioque, prioris;
> Cesare, Camestres, Festino, Baroco, secundae;
> tertia, Darapti, Disamis, Datisi, Felapton,
> Bocardo, Ferison, habet; quarta insuper addit
> Bramantip, Camenes, Dimaris, Fesapo, Fresison.

Capitalized names are valid syllogisms. Vowels are sentence forms:

A	all – is –	Aff-Irm universal/particular
I	some – is –	
E	no – is –	nE-gO universal/particular
O	some – is not –	

So "Barbara," with AAA vowels, has three "all" statements:

all M is P	MP	= figure 1
all S is M	SM	
∴ all S is P		

Aristotelian syllogisms have two premises. *Middle term* "M" is common to both premises; *predicate* "P" occurs in the first premise, while *subject* "S" occurs in the second. There are four figures (arrangements of premise letters):

1 (prioris)	2 (secundae)	3 (tertia)	4 (quarta)
MP	PM	MP	PM
SM	SM	MS	MS

Aristotle's four axioms are valid first-figure forms:

Barbara	Celarent	Darii	Ferio
all M is P	no M is P	all M is P	no M is P
all S is M	all S is M	some S is M	some S is M
∴ all S is P	∴ no S is P	∴ some S is P	∴ some S is not P

The other 15 forms can be derived as theorems. The consonants give clues on how to do this; for example, "m" says to switch the order of the premises.

Thomas Aquinas (1224–74), the most influential medieval philosopher, had little impact on logic's development; but he made much use of logic. Since he emphasized reasoning and wrote so much, he likely produced more philosophical arguments than anyone else who has ever lived.

Fourteenth-century logicians include William of Ockham and Jean Buridan. *Ockham's razor* says "Accept the simplest theory that adequately explains the data." Ockham developed modal logic and tried to avoid metaphysics when analyzing language. *Buridan's ass* was a fictional donkey whose action was paralyzed when he was placed exactly midway between two food bowls. Buridan also formulated the standard rules for valid syllogisms; one version says that a syllogism is *valid* just if it satisfies all of these conditions:

- Every term distributed in the conclusion must be distributed in the premises. (A term is *distributed* in a statement just if the statement makes some claim about *every* entity that the term refers to.)
- The middle term must be distributed in at least one premise. (The *middle term* is the one common to both premises; if we violate this rule, we commit the

fallacy of the *undistributed middle*.)

- If the conclusion is negative, exactly one premise must be negative. (A statement is *negative* if it contains "no" or "not"; otherwise it's positive.)
- If the conclusion is positive, both premises must be positive.

In the Middle Ages, logic was important in philosophy and in higher education. Even today, logic, like biology, uses many Latin terms (*modus ponens, a priori/a posteriori, de re/de dicto,* and so on).

16.3 Enlightenment logic

Aristotelian logic dominated until the end of the 19th century. Several logicians contributed to syllogistic logic; for example, Leonhard Euler diagrammed "all A is B" by putting an A-circle inside a larger B-circle, Lewis Carroll entertained us with silly syllogisms and points about logic in *Alice in Wonderland,* and John Venn gave us diagrams for testing syllogisms (§2.6). But most logicians would have agreed with Immanuel Kant, who said that Aristotle invented and perfected logic; nothing else of fundamental importance could be added, although we might improve teaching techniques. Kant would have been shocked to learn about the revolution in logic that came about a hundred years later.

The German thinkers Georg Hegel and Karl Marx provided a side current. Hegel proposed that logic should see contradictions as explaining how thought evolves historically; one view provokes its opposite, and then the two come together in a higher synthesis. Marx saw contradictions in the world as real; he applied this to political struggles and revolution. While some saw this *dialectical logic* as an alternative to traditional logic, critics objected that this confuses conflicting properties in the world (like hot/cold or capitalist/proletariat) with logical self-contradictions (like the same object being both white and, in the same sense and time and respect, also non-white).

The philosopher Gottfried Leibniz, the co-inventor of calculus, anticipated future developments. He proposed the idea of a symbolic language that would reduce reasoning to calculation. If controversies arose, the parties could take up their pencils and say, "Let us calculate." Leibniz created a logical notation much like that of Boole (and much earlier); but his work was published after Boole.

Many thinkers tried to invent an algebraic notation for logic. Augustus De Morgan proposed symbolizing "all A is B" as "A))B" and "some A is B" as "A()B"; a letter on the concave side of the parenthesis is distributed. He became known for his *De Morgan laws* for propositional logic:

Not both A and B	=	Either not-A or not-B
Not either A or B	=	Both not-A and not-B

He complained that current logic couldn't handle relational arguments like "All dogs are animals; therefore all heads of dogs are heads of animals" (§9.5b #25).

The **Boolean algebra** of George Boole (1815–64) was a breakthrough, since it used math to check the correctness of inferences. Boole used letters for sets; so "M" might be the set of mortals and "H" the set of humans. Putting two letters together represents the *intersection* of the sets; so "HM" is the set of those who are *both human and mortal*. Then "All humans are mortal" is "H = HM," which says that the set of humans = the set of those who are both human and mortal. A syllogism is a series of equations:

Valid argument	➔	All humans are mortal. All Greeks are humans. ∴ All Greeks are mortal.	H = HM G = GH ∴ G = GM

We can derive the conclusion by substituting equals for equals. In premise 2, G = GH, replace "H" with "HM" (premise 1 says H = HM) to get G = GHM. Then replace "GH" with "G" (premise 2 says G = GH) to get G = GM.

Boolean formulas, like those on the left below (which use a later symbolism), can be interpreted to be about sets or about statements:

"-A" can mean "the set of non-As" or "not-A"
"A∩B" can mean "the intersection of sets A and B" or "A and B"
"A∪B" can mean "the union of sets A and B" or "A or B"

So if "A" is the set of animals, then "-A" is the set of non-animals; but if "A" is "Aristotle is a logician," then "-A" is "Aristotle isn't a logician." The same laws cover both; for example, "A∩B = B∩A" works for either sets or statements. *Boolean operators* (like "and," "or," and "not") use the statement interpretation.

Boole, the father of *mathematical logic,* thought that logic belonged with mathematicians instead of philosophers. But both groups came to have an interest in logic, each getting the slice of the action that fits it better. While Boole was important, a greater revolution in logic was to come.

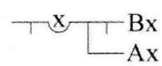
16.4 Frege and Russell

Gottlob Frege (1848–1925) created modern logic with his 1879 *Begriffsschrift* ("Concept Writing"). Its 88 pages introduced a symbolism that, for the first time, let us combine in every way Aristotle's "all," "no," and "some" with the Stoic "if-then," "and," and "or." So we can symbolize "If everything that's A or B is then C and D, then everything that's non-D is non-A." Thus the gap between Aristotle and the Stoics was overcome in a higher synthesis. Frege also showed how to analyze arguments with relations (like "x loves y") and multiple quantifiers; so we can show that "There is someone that everyone loves" entails "Everyone loves someone" – but not conversely. Frege presented logic as a *formal system*, with purely notational rules for determining the grammaticality of formulas and the correctness of proofs.

Frege's work was ignored until Bertrand Russell (1872–1970) praised it in the early 20th century. Frege's difficult symbolism alienated people. He used lines for "not," "if-then," and "all":

Not-A If A then B For all x

These can combine to symbolize "Not all A is non-B" (our "$\sim(x)(Ax \supset \sim Bx)$"):



This was also his way to write "Some A is B" (our "$(\exists x)(Ax \cdot Bx)$"); he had no simpler notation for "some" or "and."

Frege developed logic to help show that arithmetic is reducible to logic; he wanted to define all basic concepts of arithmetic (like numbers and addition) in purely logical terms and prove all basic truths of arithmetic using just logical axioms and inference rules. Frege used a seemingly harmless axiom that every condition on x picks out a set containing just those elements that satisfy that condition; so the condition "x is a cat" picks out the set of cats. But consider that some sets are members of themselves (the set of abstract objects is an abstract object) while other sets aren't (the set of cats isn't a cat). By Frege's axiom, "x is not a member of itself" picks out the set containing just those things that are not members of themselves. Call this "set R." So any x is a member of R, if and only if x is not a member of x (here "$\in$" means "is a member of" and "$\notin$" means "is not a member of"):

$$\text{For all x, } x \in R \text{ if and only if } x \notin x.$$

Russell asked in a 1902 letter to Frege: What about set R itself? By the above principle, R is a member of R, if and only if R is not a member of R:

$$R \in R \text{ if and only if } R \notin R.$$

So is R a member of itself? If it is, then it isn't – and if it isn't, then it is; either way we get a contradiction. Since this contradiction, called **Russell's paradox**, was provable in Frege's system, that system was flawed. Frege was crushed, since his life work collapsed. His attempts to fix the problem weren't successful.

Russell greatly admired Frege and his groundbreaking work in logic; the two minds worked along similar lines. But the paradox showed that Frege's work needed fixing. So Russell, with his former teacher Alfred North Whitehead, worked to develop logic and set theory in a way that avoided the contradiction. They also developed a more intuitive symbolism (much like what we use in this book), based on the work of Giuseppe Peano. The result was their massive *Principia Mathematica,* which was published in 1910–1913. *Principia* had a huge influence and became the standard formulation of the new logic.

16.5 After *Principia*

Classical symbolic logic includes propositional and quantificational logic (Chapters 6 to 9). A logic is "classical" if it accords with Frege and Russell about which arguments are valid, regardless of differences in symbolization and proof techniques. Classical symbolic logic gradually became the new orthodoxy, replacing the older Aristotelian logic.

Much work was done to solidify classical symbolic logic. Different proof techniques were developed; while Frege and Russell used an axiomatic approach, later logicians invented inferential and truth-tree methods that were easier to use. Different ways of symbolizing arguments were developed, including the *Polish notation* of a school of logic that was strong in Poland between the world wars. Ludwig Wittgenstein and Emil Post independently invented truth tables, which clarified our understanding of logical connectives (like "if-then," "and," and "or") and led to a criterion of validity based on semantics – on the meaning of the connectives and how they contribute to truth or falsity; Alfred Tarski and others expanded the semantic approach to quantificational logic.

Much work was done in **metalogic**, the study of logical systems (Chapter 15). Kurt Gödel showed that Russell's axiomatization of classical logic was, given certain semantic assumptions, correct: just the right things were provable. But he also showed, against Frege and Russell, that arithmetic cannot be reduced to any formal system: no consistent set of axioms and inference rules would suffice to prove all arithmetic truths; this result, called **Gödel's theorem**, is perhaps the most striking and surprising result of 20th-century logic. Alonzo Church showed that the problem of determining validity in quantificational logic cannot be reduced to an mechanical algorithm (a result called *Church's theorem*). There was also much activity in *set theory,* which after Russell's paradox became increasingly complex and controversial.

There was also much work in **philosophy of logic** (Chapter 18), which deals with philosophical questions about logic, such as these: Are logical truths dependent on human conventions (so different conventions might produce different logical truths) or on the objective nature of reality (perhaps giving us the framework of any possible language that would be adequate to describe reality)? Can logic help us clarify metaphysical issues, such as what kinds of entity ultimately exist? Should we assume abstract entities (like properties and propositions) when we do logic? How can we resolve logical paradoxes (such as Russell's paradox and the liar paradox)? Are logical truths empirical or *a priori*? Does logic distort ordinary beliefs and ordinary language, or does it correct them? What is the definition and scope of logic?

Logic was important in the development of computers. The key insight here was that logical functions like "and" and "or" can be simulated electrically by *logic gates*; this idea goes back to the American logician Charles Sanders Peirce in the 1880s and was rediscovered by Claude Shannon in 1938. A computer contains logic gates, plus memory and input-output devices. Logicians like John von Neumann, Alan Turing, and Arthur Burks helped design the first large-scale electronic computers. Since logic is important for computers, in both hardware and software, it's studied today in computer science departments. So now three main departments study logic – philosophy, math, and computer science.

Logic today is also an important part of *cognitive science*, an interdisciplinary approach to thought that includes linguistics, psychology, biology (brain and sensory systems), computers (especially artificial intelligence), and other branches of philosophy (especially epistemology and philosophy of mind).

As classical symbolic logic became the orthodoxy, it started to be questioned. Two types of non-classical logic came to be. **Supplementary logics** accepted that classical logic was fine as far as it went but needed to be supplemented to deal, for example, with "necessary" and "possible." **Deviant logics** thought that classical logic was wrong on some points and needed to be changed.

The most important *supplementary logic* is modal logic, which deals with "necessary" and "possible" (Chapters 10 and 11). Ancient and medieval logicians pursued modal logic; but 20th-century logicians mostly ignored it until C. I. Lewis's work in the 1930s. Modal logic then became controversial. Willard Quine argued that it was based on a confusion; he thought logical necessity was unclear and quantified modal logic led to an objectionable metaphysics of necessary properties. There was lively debate on modal logic for many years. In 1959, Saul Kripke presented a possible-worlds way to explain modal logic; this made more sense of it and gave it new respect among logicians. Possible worlds have proved useful in other areas and are now a common tool in logic; and several philosophers (including Alvin Plantinga) have defended a metaphysics of necessary properties. Today, modal logic is a well-established extension of classical logic.

Other extensions apply to ethics ("A ought to be done" or "A is good"), theory of knowledge ("X believes that A" or "X knows that A"), the part–whole relationship ("X is a part of Y"), temporal relationships ("It will be true at some future time that A" and "It was true at some past time that A"), and other areas (Chapters 12 to 14). Most logicians would agree that classical logic needs to be supplemented in order to cover certain kinds of argument.

Deviant logics say that classical symbolic logic is wrong on some points and needs to be changed (Chapter 17). Some propose using more than two truth values. Maybe we need a third truth value for "half-true." Or maybe we need a fuzzy-logic range of truth values, from completely true (1.00) to completely false (0.00). Or perhaps "A" and "not-A" can both be false (intuitionist logic) or both be true (paraconsistent logic). Or perhaps the classical approach to "if-then" is flawed; some views even reject *modus ponens* ("If A then B, A ∴ B") and *modus tollens* ("If A then B, not-B ∴ not-A"). These and other deviant logics have been proposed. Today there is much questioning of basic logical principles.

This brief history of logic has focused on deductive logic and related areas. There has also been much interest in informal logic (Chapters 3 and 4), inductive logic (Chapter 5), and history of logic (this chapter).

So logic has a complex history – from Aristotle and the Stoics in ancient Greece, through the Middle Ages and the Enlightenment, to the turmoil of the 19th century and logic's transformation with Frege and Russell, and into recent classical and non-classical logics and the birth of the computer age.[1]

[1] For more on the history of logic, I suggest P. H. Nidditch's *The Development of Mathematical Logic* (London: Routledge & Kegan Paul, 1962) and, for primary sources, Irving Copi and James Gould's *Readings on Logic* (New York: Macmillan, 1964). Also useful are William and Martha Kneale's *The Development of Logic* (Oxford: Clarendon, 1962) and Joseph Bocheński's *A History of Formal Logic*, trans. Ivo Thomas (Notre Dame, Ind.: University of Notre Dame, 1961).